International Journal of Scientific & Engineering Research, Volume 4, Issue 12, December-2013 724
ISSN 2229-5518

Comparative Analysis of Two-Channel QMF Bank Designed by new Class of Adjustable Window Functions

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Abstract— This paper proposes an efficient method for the design of Pseudo Quadrature mirror filter (QMF) bank. Parent filter of the filter bank is designed using new class of adjustable window functions. Filter coefficients are optimized by varying the cutoff frequency. A comparative analysis is included to confirm the validity of the proposed work.

Index Terms— Finite impulse response filter, Quadrature mirror filter bank, Optimization, Adjustable window functions, Near perfect reconstruction.

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1 Introduction

Quadrature mirror filter banks have received much attention since they are successfully used in the design of multirate signal processing system. These filterbanks find wide applications in many fields of signal processing such as sub-band coding of speech and image signals [1],[2],[3],[4], speech and image compression [5],[6]. Because of such wide application, researchers are giving more attention towards the efficient design of filterbanks [7],]8],[9],[10],[11],[12],[13],[14],[15], which was first introduced by Johnston [16]. In QMF bank as shown in Fig. 1 the input signal $x(n)$ splits into equally spaced frequency sub-bands using a pair of filter comprising lowpass and highpass analysis filters $H_0(z)$ and $H_1(z)$ respectively, followed by a two fold decimators to down sample the sub-band signals. At the receiving end corresponding synthesis bank has two fold interpolator for both the sub-band signals followed by $G_0(z)$ and $G_1(z)$ synthesis filters. The outputs of synthesis filters are then combined to obtain the reconstructed signal y(n). The reconstructed output signal y(n) is distorted due to aliasing ,amplitude and phase distortions. These distortions can be completely eliminated in Perfect Reconstruction case ideally but practically it is not possible .Aliasing and phase distortion has been completely eliminated by designing all the analysis/synthesis FIR linear phase filters by a single lowpass prototype even order, symmetric, linear phase, finite impulse response filter. Amplitude distortion can be minimize, but can not be eliminated completely.

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Thus researcher's main attention lies on minimization of amplitude distortion which needs optimization of certain parameter hence optimization technique have been developed. Design methods [8],[9] developed so far involve minimizing an error function directly in the frequency domain or time domain to achieve the design requirements. In the conventional QMF design technique [14],[15],[16],[17],[18],[19] to get minimum point analytically, the objective function, is evaluated by discretization, or iterative least squares methods are used which are based on the linearization of the error function to, modify the objective function. Thus, the performance of the QMF bank designed degrades as the solution obtained is the minimization of the discretized version of the objective function rather than the objective function itself, or computational complexity increased. Various design techniques including optimization based [20], and non optimization based techniques have been reported in literature for the design of QMF bank. In optimization based technique, the design problem is formulated either as multi-objective or single objective nonlinear optimization problem, which is solved by various existing methods such as least square technique, weighted least square (WLS) technique [14],[15],[16],[17] and genetic algorithm [21]. Jain and Crochiere [9] have introduced the concept of iterative algorithm and formulated the design problem in quadratic form in time domain. Thereafter, several new iterative algorithms [10],[12],[13],[14],[15],[16],[17],[18],[19],[20],[21] have been developed either in time domain or frequency domain.
In continuation, in this work a single variable linear iterative optimization algorithm proposed in [22] has been used to minimize reconstruction error near to perfect reconstruction. Kaiser, D.C, Cosh, Modified Cosh and Exponential Window functions [23],[24],[25] are used to design even order symmetric, linear phase, lowpass prototype FIR filter for pseudo QMF bank. Finally comparative evaluations of all the filters design

by above mentioned window functions have been done in terms of minimum stopband attenuation, farend attenuation, reconstruction error at fixed filter length and at fixed stopband attenuation.

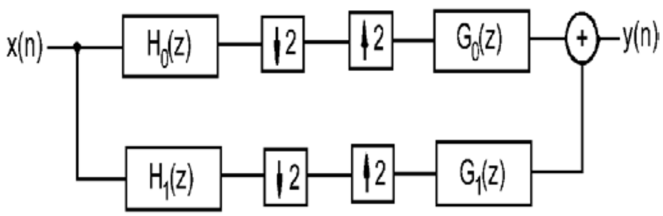


Fig. 1. Two-band QMF system

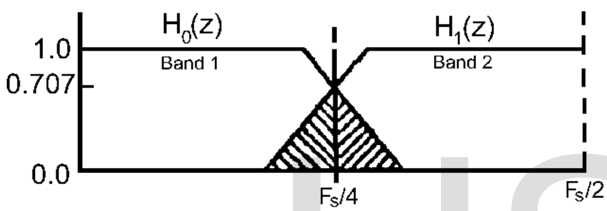


Fig. 2. Frequency response of two-band QMF system

2 ANALYSIS OF THE TWO-CHANNEL QMF BANK

The most efficient way for designing and implementing these systems is to first start with a linear phase finite impulse response (FIR) prototype filter $H(z)$. Let

$$H_0(z) = \sum_{n=0}^{N-1} h(n) z^{-n} \text{ , with } h(n) = h(N-n) \tag{1}$$

Where, $h(n)$ is the impulse response coefficients of a causal N^{th}-order linear phase FIR filter obtained using window technique as given by :

$$h(n) = w(n) h_i(n) \tag{2}$$

where $h_i(n)$ is the impulse response of the ideal low pass filter and is expressed as:

$$h_i(n) = \frac{\sin(\omega_C(n-0.5N))}{\pi(n-0.5N)} \tag{3}$$

where, ω_C is cut-off frequency of the ideal low pass filter and $w(n)$ is the window function. The prototype filter is designed using well known Kaiser, DC window functions and newly reported window functions such as Cosh , Modified Cosh and Exponential reported in [23],[24],[25] .

The *z*-transform of the output signal $y(n)$ of the two channel QMF bank, can be written as [18],[19],[20], [26]:

$$Y(z) = \frac{1}{2}\left[H_0(z)G_0(z) + H_1(z)G_1(z)\right]X(z) + \frac{1}{2}\left[H_0(-z)G_0(z) + H_1(-z)G_1(z)\right]X(-z) \tag{4}$$

The first term in above equation represents the input/output relation of the overall analysis synthesis filter bank without aliasing and imaging effects. The second term represents the effects of aliasing and imaging. The alias-free two-channel filter bank is obtained by proper combination of the transfer functions of the filters in the analysis and synthesis parts, *i.e.*,

$H_0(z), H_1(z), G_0(z)$ and $G_1(z)$.

Aliasing can be removed completely by defining the synthesis filters as given below [1, 20,21,26]:

$$G_0(z) = 2H_1(-z) \text{ and } G_1(z) = -2H_0(-z) \tag{5}$$

By using the relationship $H_1(z) = H_0(-z)$ between the mirror image filters, the expression for the distortion transfer function of the alias free QMF bank can be written as:

$$T(z) = H_0^{\,2}(z) - H_1^{\,2}(z) \tag{6}$$

It is apparent that the amplitude and phase distortion of the overall QMF bank depend on the performances of the lowpass filter $H_0(z)$. If $H_0(z)$ has a linear phase, the phase distortion of the overall filter bank is eliminated. It is shown that for $H_0(z)$ being a linear phase FIR filter of the length N, the overall frequency response $T(e^{j\omega})$ can be expressed in the following manner:

$$T(e^{j\omega}) = e^{-j\omega(N-1)}\left[\left|H_0\left(e^{j\omega}\right)\right|^2 - (-1)^{(N-1)}\left|H_1\left(e^{j\omega}\right)\right|^2\right]. \tag{7}$$

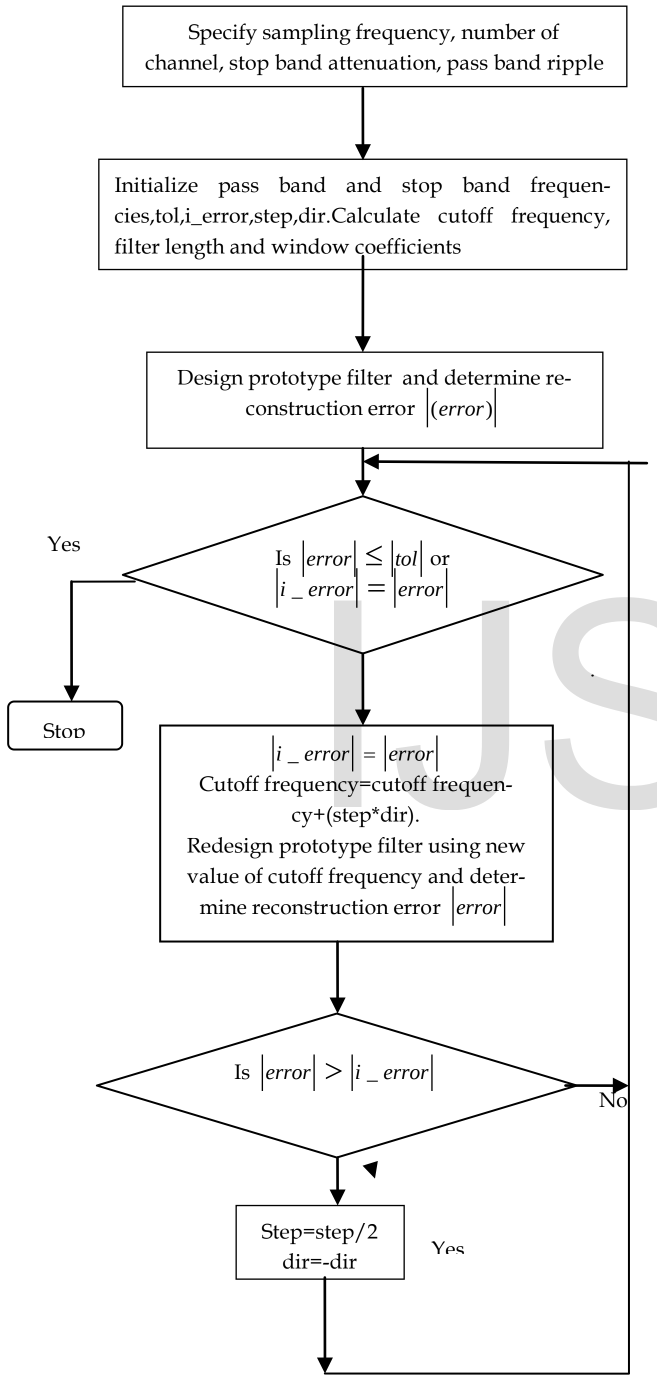
Since the filter pair $[H_0(z), H_1(z)]$ is a halfband filter pair, their magnitude responses at the cross over frequency $\omega_c = \pi/2$ are equal as shown in Fig. 2. This implies that for odd values of N, $T(e^{j\omega})$ may have severe amplitude distortions in the vicinity of $\omega_c = \pi/2$. Therefore, when using the linear phase FIR filters, the length N should be an even number. With even filter order equation (4) can be reduces to

$$T(e^{j\omega}) = e^{-j\omega(N-1)}\left[\left|H_0\left(e^{j\omega}\right)\right|^2 + \left|H_1\left(e^{j\omega}\right)\right|^2\right]. \tag{8}$$

According to equation (8), the QMF bank with linear phase filters has no phase distortion, but the amplitude distortion will always exist except for the trivial first order case. It is an important approximation problem to adjust the coefficients of the $H_0(z)$ that simultaneously provide the selective frequency responses of the analysis/synthesis filters and guarantee a small reconstruction error. A computer-aided optimization method can be employed, which iteratively adjusts the coefficients of $H_0(z)$ to achieve

$$\left|H_0\left(e^{j\omega}\right)\right|^2 + \left|H_1\left(e^{j\omega}\right)\right|^2 = 1 \tag{9}$$

The optimization algorithm discussed in [22] is applied to vary the cutoff frequency of the lowpass prototype filter. The reconstruction error is selected as the objective function.
The objective function given in equation. (9) has been used to minimize the reconstruction error to near perfect reconstruc-



tion. In the optimization algorithm cutoff frequency (ω_c) is varied to get the smallest value of reconstruction error. The algorithm adjusts the cutoff frequency (ω_c) by step size in each iteration , calculates the new filter coefficients , computes the reconstruction error, compare it with previous error, accordingly step size and search direction has been changed. The iterations are ss the error of present iteration is within the specified tolerance initialized previously or no improvement has been made from the previous value. The proposed technique has been implemented in MATLAB and its flow graph is shown in Fig. 3 given below.

3 CASE STUDY AND PERFORMANCE ANALYSIS

Under the case study we consider two examples, we choose the Johnston's FIR filters of the length N = 24 and 32 to examine the performances of a linear phase FIR QMF bank. We determine first the analysis filters $H_0(z)$ and $H_1(z)$, and plot their characteristics in frequency domain. Determine reconstruction error and minimize it by varying cutoff frequency of the prototype filter.

Example 1-

With N =24, ω_p =0.4π, ω_s = 0.6π, function *tolerance*=1e-6, *initial error* = 250 *dir* = 1, *step* = 0.05 a QMF bank has been designed by windowed FIR filter.
The significant parameters obtained are listed in **TABLE 1.**

TABLE 1: Performance parameters.

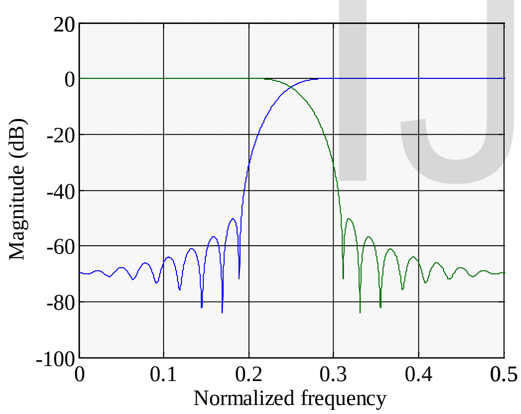
Window function	Minimum stop-band Attenuation (A_s) dB	Peak reconstruction error (E_p)	Far end Attenuation (A_f)(dB)	Phase response
DC	-42.4	0.0242 dB	-52.07	Linear
Kaiser	-42.4	0.0800 dB	-53.00	Linear
Cosh	-40.6	0.1040 dB	-89.97	Linear
Modified cosh	-30.7	0.0515 dB	-66.66	Linear
Exponential	-38.8	0.1417 dB	-52.42	Linear

Example 2-

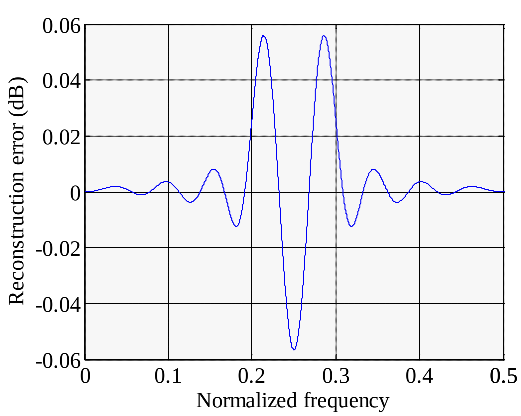
For N = 32, ω_p = 0.4π, ω_s = 0.6π, performance parameters are listed in **TABLE 2.**The corresponding normalized magnitude plots of the analysis filters $H_0(z)$ and $H_1(z)$ are shown in Fig. 4 (a,c,e). Fig. 4 (b,d,f) shows the peak reconstruction error of the QMF bank . The significant parameters obtained are: E_p = 0.0052, A_s = **-42.415** dB, A_f = -99.53 dB and actual value of stopband attenuation is -86.93 dB for Modified Cosh window based design with multiplicative factor $\rho = 7$.
The simulation results of the proposed method are compared with the Gradient method [27], General purpose method [24], Smith Barnwell method [15], Jain Crochiere [9], Chen Lee [8], Xu-Lu Antoniou [29], Lu-Xu Antoniou [21] and Sahu *et al.* [17],

ISSN 2229-5518

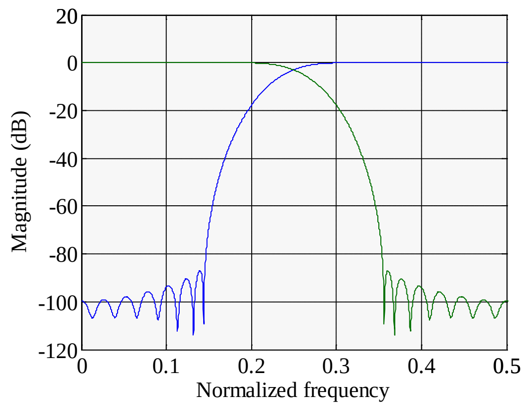
for N = 32, and are summarized in **TABLE 2**. The results show that Modified Cosh window based design performs better than Gradient, General purpose, Smith Barnwell, Jain Crochiere, Chen-Lee, Xu-Lu-Antoniou, Lu-Xu-Antoniou and Sahu methods in terms of peak reconstruction error E_p and far end stop band attenuation A_f. It gives much better minimum stop band attenuation than [8],[9],[17],[21],[22],[27]. The obtained actual value of stop band attenuation as shown in Fig. 4(c) is highest amongst all the proposed windows. According to the results obtained, some observation about filter characteristics is also made. Variation of peak reconstruction error with filter length at constant A_s is shown in Fig. 5. It is observed that Modified Cosh window based design shows superior performance. Similarly from Fig. 6 it is observed that peak reconstruction error decreases as stopband attenuation increases for Kaiser, DC, Cosh and Exponential windows. For Modified Cosh window peak reconstruction error increases with increasing values of stopband attenuation. At most it is observed that by introducing a third parameter (p) in the window function a better window function can be obtained for FIR filter design where higher main-lobe width and smaller ripple ratio is important. It also leads to better side-lobe roll off ratio. Thus it gives a better response over cosh window, Kaiser window, DC and Exponential window



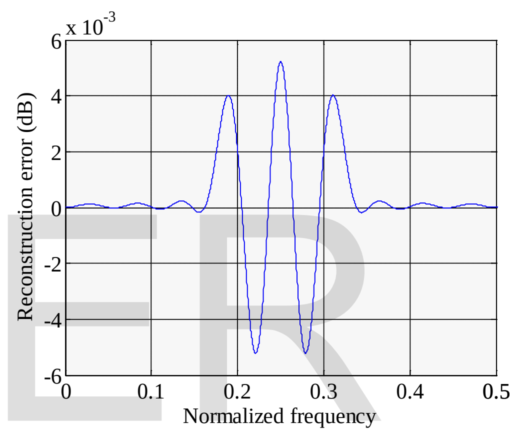
4(a) Frequency response plot (Cosh)



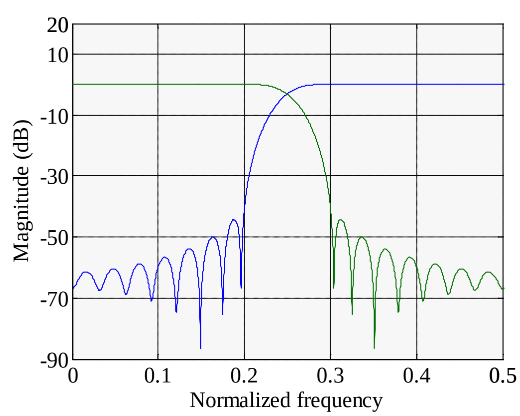
4(b) Reconstruction error plot (Cosh)



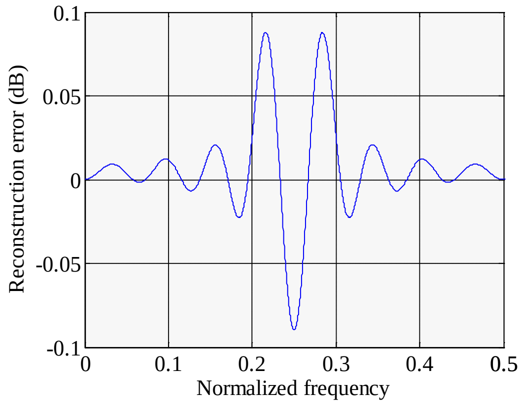
4(c) Frequency response plot (Modified Cosh)



4(d) Reconstruction error plot (Modified Cosh)



4(e) Frequency response plot (Exponential)



4(f) Reconstruction error plot (Exponential)

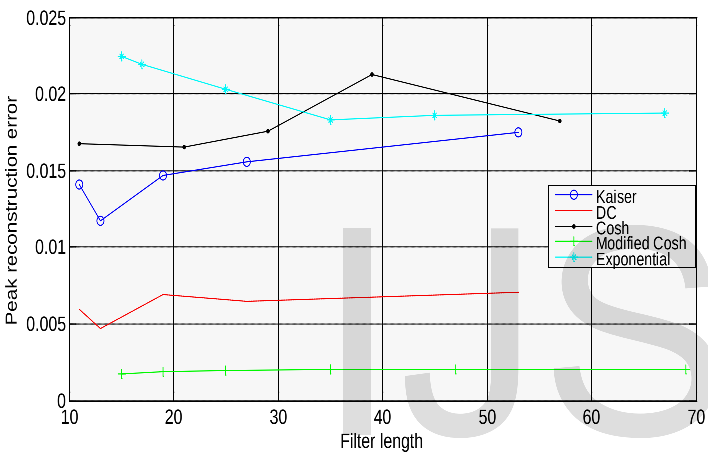


Fig. 5 Filter length versus Peak reconstruction error plot

TABLE 2: Performance comparison of proposed QMF bank with previous work.

Work	N	A_s (dB)	E_p (dB)	A_f (dB)	Phase response
Gradient method [27]	32	-33.6	0.009	-	Nonlinear
General purpose [28]	32	-49.2	0.016	-	Linear
Smith–Barnwell [15]	32	-39.0	0.019		Nonlinear
Jain-Crochiere[9]	32	-33.0	0.015	-54	Linear
Chen-Lee[8]	32	-34.0	0.016	-52	Linear
Xu-Lu-Antoniou[29]	32	-35.0	0.031	-54	Linear
Lu-Xu-Antoniou[21]	32	-35.0	0.015	--	Linear
Sahu *et al.*[17]	32	-33.913	0.0269	--	Linear
Proposed *DC*	32	-53.90	0.01048	-53.9	Linear
Proposed *Kaiser*	32	-53.90	0.04244	-61.04	Linear
Proposed *Cosh*	32	-50.50	0.0532	-74.01	Linear
Proposed *Modified – cosh*	32	-42.41	0.0052	-99.53	Linear
Proposed *Exponential*	32	-44.5	0.0879	-63.45	Linear

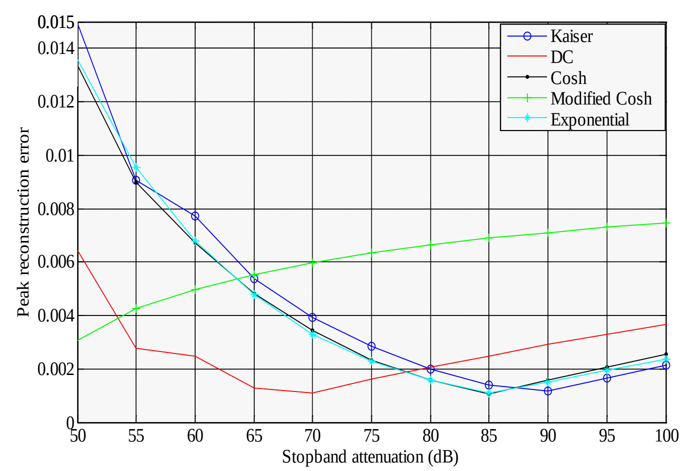


Fig. 6 Stopband attenuation versus Peak reconstruction error plot

4 CONCLUSION

New variable windows have been proposed for designing the low-pass prototype filters for QMF banks. Linear iterative optimization algorithm has been use to optimize the filter coefficients to get minimum value of reconstruction error by varying the filter cutoff frequency. The Modified Cosh window based design showed optimum performance in terms of reconstruction error and far end attenuation at the cost of increased arithmetic complexity. Better far end rejection feature helps to reduce the aliasing energy leak into a sub band from that of the signal in the other sub band.

5 ACKNOWLEDGEMENT

The corresponding author wish to thank Dr. Alok Jain of the same institute for his valuable and constructive suggestions.

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